McDougal Littell

ALGEBRA 2

EQUATIONS

APPLICATIONS

GRAPHS

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FOCUS ON PEOPLE

GREGORY ROBERTSON made a daring rescue while skydiving in 1987. To reach a novice skydiver in trouble, he increased his velocity by diving headfirst towards the other diver. Just 10 seconds before he would have hit the ground, he was able to deploy both of their chutes.

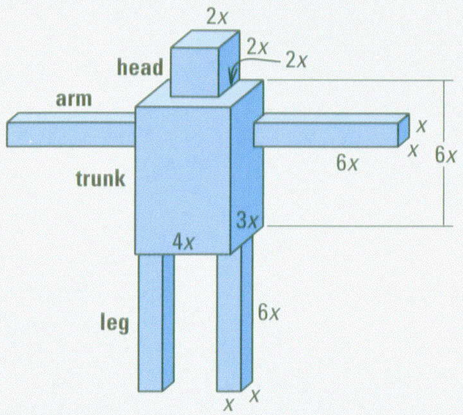
GOAL 2 USING RATIONAL EXPRESSIONS IN REAL LIFE

EXAMPLE 8 *Writing and Simplifying a Rational Model*

SKYDIVING A falling skydiver accelerates until reaching a constant falling speed, called the *terminal velocity*. Because of air resistance, the ratio of a skydiver's volume to his or her cross-sectional surface area affects the terminal velocity: the larger the ratio, the greater the terminal velocity.

a. The diagram shows a simplified geometric model of a skydiver with maximum cross-sectional surface area. Use the diagram to write a model for the ratio of volume to cross-sectional surface area for a skydiver.

b. Use the result of part (a) to compare the terminal velocities of two skydivers: one who is 60 inches tall and one who is 72 inches tall.



SOLUTION

a. The volume and cross-sectional surface area of each part of the skydiver are given in the table below. (Assume that the front side of the skydiver's body is parallel with the ground when falling.)

Body part	Volume	Cross-sectional surface area
Arm or leg	$V = 6x^3$	$S = 6x(x) = 6x^2$
Head	$V = 8x^3$	$S = 2x(2x) = 4x^2$
Trunk	$V = 72x^3$	$S = 6x(4x) = 24x^2$

Using these volumes and cross-sectional surface areas, you can write the ratio as:

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{4(6x^3) + 8x^3 + 72x^3}{4(6x^2) + 4x^2 + 24x^2}$$

$$= \frac{104x^3}{52x^2}$$

$$= 2x$$

b. The overall height of the geometric model is $14x$. For the skydiver whose height is 60 inches, $14x = 60$, so $x \approx 4.3$. For the skydiver whose height is 72 inches, $14x = 72$, so $x \approx 5.1$. The ratio of volume to cross-sectional surface area for each skydiver is:

60 inch skydiver: $\frac{\text{Volume}}{\text{Surface area}} = 2x \approx 2(4.3) = 8.6$

72 inch skydiver: $\frac{\text{Volume}}{\text{Surface area}} = 2x \approx 2(5.1) = 10.2$

▶ The taller skydiver has the greater terminal velocity.